

A note on the role of the boundary terms for the non-Gaussianity in general k-inflation

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In this short note we clarify the role of the boundary terms in the calculation of the leading order tree-level bispectrum in a fairly general minimally coupled single field inflationary model, where the inflaton's Lagrangian is a general function of the scalar field and its first derivatives. This includes k-inflation, DBI-inflation and standard kinetic term inflation as particular cases. These boundary terms appear when simplifying the third order action by using integrations by parts. We perform the calculation in the comoving gauge obtaining explicitly all total time derivative interactions and show that a priori they cannot be neglected. The final result for the bispectrum is equal to the result present in the literature which was obtained using the field redefinition.

I. INTRODUCTION

The inflationary scenario has become the most compelling idea for what really happened in the very early stages of our universe. The main reason for this is that inflation is very successful at explaining the problems of the old Big Bang model and it also provides us with a mechanism to generate the primordial seed perturbations that later evolved into the cosmic microwave background radiation anisotropies and the large-scale structure of galaxies.

The most simple models of inflation predict a nearly Gaussian and nearly scale invariant primordial perturbation in good agreement with recent observations. However, a small amount of non-Gaussianity is still allowed by the data. If such a small contribution were detected and if it were of primordial origin, it would have profound implications for inflationary models and our understanding of the very early universe. Because of this reason, recently there has been a huge effort to try to construct models that predict large (i.e. observable) non-Gaussianity and calculate the higher-order correlation functions like the bispectrum and the trispectrum. This search has been productive. Many possibilities have been found, for example models with non-canonical kinetic terms (like DBI-inflation, k-inflation, ghost-inflation) [1–12], multiple field models of inflation [13–44], temporary violations of the slow-roll conditions and small departures of the initial vacuum state from the standard Bunch-Davies vacuum [45–51]. For recent reviews about these mechanisms to produce non-Gaussian perturbations, see Ref. [52–56].

In this note, we will be interested in the calculation of the bispectrum for quite general models of k-inflation. Our conclusions will apply to all k-inflation, DBI-inflation and standard kinetic term inflation models. Our main goal is to clarify the role of the boundary terms in the calculation of the bispectrum. These boundary interactions appear when one does many integrations by parts to simplify the third order action.

For the standard kinetic term inflation all these boundary terms have been for the first time recently calculated in [57]. However in all calculations of the bispectrum so far these terms have been neglected and a field redefinition was used instead. Maldacena [58] states that these boundary interactions are important and he takes them into account using field redefinitions. We should point out one exception [59], where the authors working in the uniform curvature gauge and considering standard kinetic term inflation show explicitly that the bispectrum can be determined by using the third order action without the need to redefine the field if the contribution from the boundary terms is included.

In this work, we will perform an analogous calculation, but this time for a general k-inflation model and working in the comoving gauge. In this gauge, one works from the beginning with the variable of interest, the comoving curvature perturbation. Naively, the third order action does not seem to be suppressed by the slow-roll parameters contrarily to the expected. However it has been shown [4, 58] that after many integrations by parts this suppression becomes evident. This procedure produces many boundary terms which are the main focus of this work.

This paper is organized as follows. In section II, we will introduce the model, the background spacetime and some useful notation. In section III, we will discuss linear perturbations and present the solution for the mode functions. In section IV, we shall review the usual calculation of the third order action and the bispectrum and obtain the new boundary terms which we argue can be used to calculate the bispectrum without the need to do a field redefinition.

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We also present the calculation of the bispectrum produced by these time boundary interactions. Section V is devoted to the conclusions.

II. THE MODEL

In this note, we will consider the class of models described by the following Lagrangian

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_{Pl}^2 R + 2P(X, \phi)], \quad (1)$$

where ϕ is the inflaton field, M_{Pl} is the reduced Planck mass, R is the Ricci scalar, $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is the inflaton's kinetic energy and $g_{\mu\nu}$ is the metric tensor. $P(X, \phi)$ denotes the inflaton's Lagrangian and we assume it is a well-behaved function of its two variables. Throughout this work, we use a system of units where the Planck constant \hbar , the speed of light c and the reduced Planck mass M_{Pl} are set to unity. This general Lagrangian includes as particular cases the DBI-inflation model [2, 60] and the k-inflation model [61].

We are interested in flat, homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker background universes described by the line element

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (2)$$

where $a(t)$ is the scale factor. The Friedmann equation and the continuity equation read

$$3H^2 = \rho, \quad \dot{\rho} = -3H(\rho + P), \quad (3)$$

where dot denotes derivative with respect to cosmic time, the Hubble rate is $H = \dot{a}/a$, ρ is the energy density of the inflaton and it is given by

$$\rho = 2XP_{,X} - P, \quad (4)$$

where $P_{,X}$ denotes the derivative of P with respect to X . It was shown in [62] that for this model the speed of propagation of scalar perturbations (“speed of sound”) is c_s given by

$$c_s^2 = \frac{P_{,X}}{\rho_{,X}} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}. \quad (5)$$

We define the slow-variation parameters, analogues of the slow-roll parameters, as

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{XP_{,X}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}, \quad s = \frac{\dot{c}_s}{c_s H}. \quad (6)$$

We assume that the rate of change of the speed of sound is small (as described by s) but c_s is otherwise free to change between zero and one. It is convenient to introduce the following parameters that describe the non-linear dependence of the Lagrangian on the kinetic energy

$$\Sigma = XP_{,X} + 2X^2P_{,XX} = \frac{H^2\epsilon}{c_s^2}, \quad \lambda = X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}. \quad (7)$$

These parameters are related to the size of the bispectrum.

III. PERTURBATIONS

In this section we will consider linear perturbations of the background (2). There is a vast amount of works on linear perturbations, see for example [62]. It is convenient to use the Arnowitt, Deser and Misner (ADM) metric formalism [63]. The ADM line element reads

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (8)$$

where N is the lapse function, N^i is the shift vector and h_{ij} is the 3D metric.

The action (1) becomes

$$S = \frac{1}{2} \int dt d^3x \sqrt{h} N \left({}^{(3)}R + 2P \right) + \frac{1}{2} \int dt d^3x \sqrt{h} N^{-1} (E_{ij} E^{ij} - E^2). \quad (9)$$

The tensor E_{ij} is defined as

$$E_{ij} = \frac{1}{2} \left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i \right), \quad (10)$$

and it is related to the extrinsic curvature of the spatial slices by $K_{ij} = N^{-1} E_{ij}$. ∇_i is the covariant differentiation with respect to h_{ij} and all contra-variant indices in this section are raised with h_{ij} unless stated otherwise.

The Hamiltonian and momentum constraints are respectively

$$\begin{aligned} {}^{(3)}R + 2P - 2\pi^2 N^{-2} P_{,X} - N^{-2} (E_{ij} E^{ij} - E^2) &= 0, \\ \nabla_j (N^{-1} E_i^j) - \nabla_i (N^{-1} E) &= \pi N^{-1} \nabla_i \phi P_{,X}, \end{aligned} \quad (11)$$

where π is defined as $\pi \equiv \dot{\phi} - N^j \nabla_j \phi$. We decompose the shift vector N^i into scalar and intrinsic vector parts as $N_i = \tilde{N}_i + \partial_i \psi$, where $\partial_i \tilde{N}^i = 0$, here the index is raised with δ_{ij} .

On the comoving time-slices, the scalar field fluctuations vanish, $\delta\phi = 0$, and the three-dimensional spatial metric h_{ij} is perturbed as [58]

$$h_{ij} = a^2 e^{2\zeta} \delta_{ij}, \quad (12)$$

where ζ denotes the curvature perturbation on comoving slices and tensor perturbations have been neglected because they do not contribute to the tree-level scalar bispectrum.

We expand N and N^i in powers of the perturbation ζ as

$$N = 1 + \alpha_1 + \alpha_2 + \dots, \quad \tilde{N}_i = \tilde{N}_i^{(1)} + \tilde{N}_i^{(2)} + \dots, \quad \psi = \psi_1 + \psi_2 + \dots, \quad (13)$$

where α_n , $\tilde{N}_i^{(n)}$ and ψ_n are of order ζ^n .

Now, the strategy is to solve the constraint equations for the lapse function and shift vector in terms of ζ and then plug in the solutions in the expanded action up to the desired order. It turns out that even for the third order action one does not need to use the explicit solution for the constraints past first order [4, 58].

At first order in ζ , the solution for equations (11) with a particular choice of boundary conditions at spatial infinity is [4, 58, 64]

$$\alpha_1 = \frac{\dot{\zeta}}{H}, \quad \tilde{N}_i^{(1)} = 0, \quad \psi_1 = -\frac{\zeta}{H} + \chi, \quad \partial^2 \chi = a^2 \frac{\epsilon}{c_s^2} \dot{\zeta}. \quad (14)$$

The second order action is then

$$S_2 = \int dt d^3x \frac{a^3 \epsilon}{c_s^2} \left(\dot{\zeta}^2 - a^{-2} c_s^2 (\partial \zeta)^2 \right). \quad (15)$$

At leading order in slow-roll, H is constant and the scale factor may be approximated by that of a pure de Sitter universe $a = -1/(H\tau)$ and $-Ht = \ln(-H\tau)$, where τ denotes conformal time. The solution of the equation of motion derived from the previous action at leading order in the slow-variation parameters and in Fourier space is [4]

$$\zeta(\tau, \mathbf{k}) = \frac{iH}{\sqrt{4\epsilon c_s k^3}} (1 + i k c_s \tau) e^{-i k c_s \tau}. \quad (16)$$

To quantize the curvature perturbation, we follow the standard procedure in quantum field theory. We promote ζ to an operator that is expanded in terms of creation and annihilation operators and mode functions as

$$\hat{\zeta}(\tau, \mathbf{k}) = \zeta(\tau, \mathbf{k}) a(\mathbf{k}) + \zeta^*(\tau, -\mathbf{k}) a^\dagger(-\mathbf{k}), \quad (17)$$

where a and a^\dagger satisfy the usual commutation relation $[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')$.

IV. THE THIRD ORDER ACTION AND THE BISPECTRUM

In this section, we will present the standard equations needed in the calculation of the bispectrum of the primordial curvature perturbation using the so-called in-in formalism [65, 66]. In subsection IV A we shall consider the calculation using the well-known field redefinition prescription and in subsection IV B we will show that we can obtain the same result if we include the contribution from the boundary terms that appear when we simplify the action to the form (18) by using integrations by parts.

In order to use the machinery of the in-in formalism to compute the tree-level three-point correlation function (or bispectrum) one needs to calculate the cubic-order interaction Hamiltonian, see for example [52] for a review about this procedure.

A. The field redefinition

The third order action, ignoring the many boundary terms, has been known since the seminal work by Maldacena [58] (for the standard kinetic term case) and can be also found in [4, 64] (for the model (1)), it reads

$$S_3^{Reduced} = \int dt d^3x \left[-a^3 \left(\Sigma \left(1 - \frac{1}{c_s^2} \right) + 2\lambda \right) \frac{\dot{\zeta}^3}{H^3} + \frac{a^3 \epsilon}{c_s^4} (\epsilon - 3 + 3c_s^2) \zeta \dot{\zeta}^2 + \frac{a\epsilon}{c_s^2} (\epsilon - 2s + 1 - c_s^2) \zeta (\partial\zeta)^2 \right. \\ \left. - 2a \frac{\epsilon}{c_s^2} \dot{\zeta} (\partial\zeta) (\partial\chi) + \frac{a^3 \epsilon}{2c_s^2} \frac{d}{dt} \left(\frac{\eta}{c_s^2} \right) \zeta^2 \dot{\zeta} + \frac{\epsilon}{2a} (\partial\zeta) (\partial\chi) \partial^2 \chi + \frac{\epsilon}{4a} (\partial^2 \zeta) (\partial\chi)^2 + 2f(\zeta) \frac{\delta L}{\delta \zeta} \Big|_1 \right], \quad (18)$$

where we should note that no slow-roll approximation has been made and we define

$$\chi = \frac{a^2 \epsilon}{c_s^2} \partial^{-2} \dot{\zeta}, \quad \frac{\delta L}{\delta \zeta} \Big|_1 = a \left(\frac{d\partial^2 \chi}{dt} + H \partial^2 \chi - \epsilon \partial^2 \zeta \right), \quad (19)$$

$$f(\zeta) = \frac{\eta}{4c_s^2} \zeta^2 + \frac{1}{c_s^2 H} \zeta \dot{\zeta} + \frac{1}{4a^2 H^2} [-(\partial\zeta)(\partial\zeta) + \partial^{-2}(\partial_i \partial_j (\partial_i \zeta \partial_j \zeta))] + \frac{1}{2a^2 H} [(\partial\zeta)(\partial\chi) - \partial^{-2}(\partial_i \partial_j (\partial_i \zeta \partial_j \chi))]. \quad (20)$$

In Eqs. (19) and (20), ∂^{-2} denotes the inverse Laplacian and $\delta L/\delta \zeta|_1$ denotes the variation of the quadratic action with respect to the perturbation ζ . The last term in Eq. (18) can be absorbed by a field redefinition of ζ [4],

$$\zeta \rightarrow \zeta_n + f(\zeta_n). \quad (21)$$

After this redefinition of variables, there appears a contribution to the interaction Hamiltonian at cubic order coming from the originally second-order action (See Eq. (31) below). This term is identical to the symmetric of the last term in Eq. (18) up to a surface term arising from the integration by parts. Neglecting such a surface term, the interaction Hamiltonian in conformal time becomes

$$H_{int}(\tau) = - \int d^3x \left[- \left(\Sigma \left(1 - \frac{1}{c_s^2} \right) + 2\lambda \right) \frac{\zeta_n'^3}{H^3} + \frac{a\epsilon}{c_s^4} (\epsilon - 3 + 3c_s^2) \zeta_n \zeta_n'^2 + \frac{a\epsilon}{c_s^2} (\epsilon - 2s + 1 - c_s^2) \zeta_n (\partial\zeta_n)^2 \right. \\ \left. - 2 \frac{\epsilon}{c_s^2} \zeta_n' (\partial\zeta_n) (\partial\chi_n) + \frac{a\epsilon}{2c_s^2} \left(\frac{\eta}{c_s^2} \right)' \zeta_n^2 \zeta_n' + \frac{\epsilon}{2a} (\partial\zeta_n) (\partial\chi_n) \partial^2 \chi_n + \frac{\epsilon}{4a} (\partial^2 \zeta_n) (\partial\chi_n)^2 \right], \quad (22)$$

where prime denotes derivative with respect to conformal time and $\chi_n = a\epsilon c_s^{-2} \partial^{-2} \zeta_n'$.

The field redefinition (21) introduces extra terms in the three-point function as

$$\langle \zeta(\mathbf{x}_1) \zeta(\mathbf{x}_2) \zeta(\mathbf{x}_3) \rangle = \langle \zeta_n(\mathbf{x}_1) \zeta_n(\mathbf{x}_2) \zeta_n(\mathbf{x}_3) \rangle \\ + \frac{\eta}{2c_s^2} \langle \zeta_n(\mathbf{x}_1) \zeta_n(\mathbf{x}_2) \rangle \langle \zeta_n(\mathbf{x}_1) \zeta_n(\mathbf{x}_3) \rangle + \text{sym} + \mathcal{O}(\eta^2 c_s^{-4} (P_\zeta)^3), \quad (23)$$

where “sym” denotes two terms that result from the preceding one by symmetrizing with respect to \mathbf{x}_1 , \mathbf{x}_2 and \mathbf{x}_3 . The field redefinition (21) includes several other terms, however in the previous expression we only displayed the contribution of the first term in (20). This is because the omitted terms involve at least one derivative of ζ and they

should vanish when evaluated outside the horizon giving a negligible contribution to (23). For the reason mentioned above, in this paper we approximate the function $f(\zeta)$ as

$$f(\zeta) \approx \frac{\eta}{4c_s^2} \zeta^2. \quad (24)$$

In the expression (23), the slow roll parameter η has to be evaluated at the end of inflation. This means that η might become large depending on how inflation ends which in turn would imply that the expansion in terms of η would cease to make sense. However in most cases one can safely ignore the last term in Eq. (23), which is higher order in η because this is also greatly suppressed by powers of the spectrum P_ζ which is measured to be of order 10^{-10} .

The tree-level three-point correlation function at the time τ_e after horizon exit is

$$\langle \Omega | \hat{\zeta}_n(\tau_e, \mathbf{k}_1) \hat{\zeta}_n(\tau_e, \mathbf{k}_2) \hat{\zeta}_n(\tau_e, \mathbf{k}_3) | \Omega \rangle = -i \int_{-\infty}^{\tau_e} d\tilde{\tau} a \langle 0 | [\hat{\zeta}_n(\tau_e, \mathbf{k}_1) \hat{\zeta}_n(\tau_e, \mathbf{k}_2) \hat{\zeta}_n(\tau_e, \mathbf{k}_3), \hat{H}_{int}(\tilde{\tau})] | 0 \rangle, \quad (25)$$

where $|\Omega\rangle$ and $|0\rangle$ denotes the interacting vacuum and the free theory vacuum respectively. $[\cdot, \cdot]$ denotes the standard commutator and the interaction Hamiltonian \hat{H}_{int} is used to evolve the free theory vacuum to the interaction vacuum at the time when the three-point function is evaluated [58].

In Maldacena's calculation [58] and in the following ones [4, 64] the last three terms in Eq. (22) which are higher-order in the slow-roll expansion were properly neglected because these authors work at leading order in slow-roll. In this work, we are only interested in the bispectrum produced by the field redefinition, i.e. the second line of Eq. (23). In Fourier space it reads [4]

$$\langle \Omega | \hat{\zeta}(0, \mathbf{k}_1) \hat{\zeta}(0, \mathbf{k}_2) \hat{\zeta}(0, \mathbf{k}_3) | \Omega \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{K}) \frac{H^4 \eta}{32c_s^4 \epsilon^2} \frac{k_1^3 + k_2^3 + k_3^3}{(k_1 k_2 k_3)^3}, \quad (26)$$

where \mathbf{K} is defined as $\mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$.

B. The boundary terms

In the previous expression for the third order action (18) we omitted both time and spatial boundary terms as in previous calculations. However the total action includes them and in this subsection we will obtain these interactions explicitly. In the following, total spatial derivative terms will be omitted because they do not contribute to the three point function. It is easy to see that the interaction Hamiltonian of these terms is proportional to $\mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$ which has to vanish because of momentum conservation imposed by the overall Dirac delta function.

The third order action without neglecting the boundary terms is given by

$$S_3^{Total} = S_3^{Reduced} + S_3^{Boundary}, \quad (27)$$

and the explicit form of the boundary term is

$$\begin{aligned} S_3^{Boundary} = \int dt d^3x \frac{d}{dt} \Big[& -9a^3 H \zeta^3 + \frac{a}{H} \zeta (\partial \zeta)^2 - \frac{1}{4aH^3} (\partial \zeta)^2 \partial^2 \zeta \\ & - \frac{a\epsilon}{c_s^2 H} \zeta (\partial \zeta)^2 - \frac{\epsilon a^3}{c_s^4 H} \zeta \dot{\zeta}^2 + \frac{1}{2aH^2} \zeta (\partial_i \partial_j \zeta \partial_i \partial_j \chi - \partial^2 \zeta \partial^2 \chi) \\ & - \frac{\eta a}{2c_s^2} \zeta^2 \partial^2 \chi - \frac{1}{2aH} \zeta (\partial_i \partial_j \chi \partial_i \partial_j \chi - \partial^2 \chi \partial^2 \chi) \Big], \end{aligned} \quad (28)$$

where once again total spatial derivative terms were omitted. One can calculate the three point functions coming from the individual terms in this expression, directly using ζ . The difference in the third order action compared with the case of ζ_n is the last term in (18) and $S_3^{Boundary}$. The last term in (18), $2f(\zeta)(\delta L/\delta \zeta)|_1$ does not give any contribution because the factor $(\delta L/\delta \zeta)|_1$ vanishes after substitution of the mode function. The contributions from the different terms in $S_3^{Boundary}$ all vanish except for the one coming from the first term in the third line (using the leading order mode function and taking the limit $\tau_e \rightarrow 0$).

The contribution from the first term in the third line, i.e.

$$\int dt d^3x \frac{d}{dt} \left(-\frac{\eta \epsilon a^3}{2c_s^4} \zeta^2 \dot{\zeta} \right), \quad (29)$$

can be calculated using the in-in formalism to find

$$\begin{aligned}
& \langle \Omega | \hat{\zeta}(\tau_e, \mathbf{k}_1) \hat{\zeta}(\tau_e, \mathbf{k}_2) \hat{\zeta}(\tau_e, \mathbf{k}_3) | \Omega \rangle \\
&= (2\pi)^3 \delta^{(3)}(\mathbf{K}) \Im \left[\eta \epsilon c_s^{-4} a^2 \zeta(\tau_e, \mathbf{k}_1) \zeta(\tau_e, \mathbf{k}_2) \zeta(\tau_e, \mathbf{k}_3) \zeta^*(\tau_e, \mathbf{k}_1) \zeta^*(\tau_e, \mathbf{k}_2) \zeta'^*(\tau_e, \mathbf{k}_3) \right] \Big|_{\tau_e \rightarrow 0} + 5 \text{ perms.} \\
&= (2\pi)^3 \delta^{(3)}(\mathbf{K}) \frac{H^4 \eta}{32 c_s^4 \epsilon^2} \frac{k_1^3 + k_2^3 + k_3^3}{(k_1 k_2 k_3)^3},
\end{aligned} \tag{30}$$

where we used the mode function solution (16). This bispectrum is of the so-called local type and exactly agrees with Eq. (26). This result shows explicitly that the total time derivative terms cannot a priori be neglected [58, 59]. The total bispectrum is the sum of the expression (30) (or equivalently (26)) with the bispectrum $\langle \zeta_n(0, \mathbf{k}_1) \zeta_n(0, \mathbf{k}_2) \zeta_n(0, \mathbf{k}_3) \rangle$, which is calculated using Eq. (25). This is the result that Chen *et al.* [4] got. They performed the calculation by the field redefinition prescription and used Wick's theorem as described in the preceding subsection.

In the approach of using the field redefinition explained in the preceding section, we did not take into account the boundary action S_3^{Boundary} . If we perform the change of variables $\zeta = \zeta_n + f(\zeta_n)$ without neglecting the surface term, it generates a third order action from the second order action (15) as

$$\begin{aligned}
S_3^{CV} &= \int dt d^3x (-2) f(\zeta_n) \frac{\delta L}{\delta \zeta_n} \Big|_1 + \int dt d^3x \frac{d}{dt} \left(2a^3 \frac{\epsilon}{c_s^2} f(\zeta_n) \dot{\zeta}_n \right) \\
&\approx \int dt d^3x (-2) f(\zeta_n) \frac{\delta L}{\delta \zeta_n} \Big|_1 + \int dt d^3x \frac{d}{dt} \left(\frac{a^3 \epsilon \eta}{2c_s^4} \zeta_n^2 \dot{\zeta}_n \right),
\end{aligned} \tag{31}$$

where again total spatial derivative terms were omitted because they do not contribute to the three point function, and the approximate equality “ \approx ” means the truncation given in Eq. (24), e.g. we neglect total time derivatives terms that do not contribute to the leading order three point function. The first and second terms, respectively eliminate the last terms in S_3^{Reduced} proportional to the linear equations of motion and the boundary term that contributes to the bispectrum.

The boundary terms (28) are necessary to erase from the action the terms with second order time derivative on ζ contained in the last interaction in Eq. (18). Without these boundary terms, the second order time derivative terms can remain in the action. These problematic interactions from the point of view of the Hamiltonian formalism are not present in the original action Eq. (9), and they are not generated by the procedure of replacing the solution of the constraints back into the action. In fact, these terms are generated by the integrations by parts, and therefore the inclusion of the boundary terms naturally takes care of them.

The calculations presented above clearly show that the boundary terms can affect the result of the computation. However, if the results of the computation depend on the choice of the total derivative terms in the action $\int dt d^3x d\mathcal{B}(\zeta, \dot{\zeta})/dt$, one might be worried of how one can correctly choose \mathcal{B} . The terms in \mathcal{B} containing $\dot{\zeta}$ produce the second order time derivatives in the action. As mentioned above, such problematic terms should vanish in total. This condition completely determines the terms containing $\dot{\zeta}$ in \mathcal{B} . In contrast, there is no criteria to choose the terms that do not contain $\dot{\zeta}$. However, those terms do not contribute to the equal-time expectation values of ζ s. In the path integral expression for the expectation value of an operator $\mathcal{O}(t)$, adding such boundary terms given by $\mathcal{B}(\zeta)$ is equivalent to replacing the operator \mathcal{O} with $\mathcal{O}' \equiv e^{-i \int d^3x \mathcal{B}(\zeta)} \mathcal{O} e^{i \int d^3x \mathcal{B}(\zeta)}$. However, since $\mathcal{B}(\zeta)$ does not contain the conjugate momentum of ζ , \mathcal{B} commutes with \mathcal{O} . Therefore \mathcal{O}' reduces to \mathcal{O} and hence the results are not affected by such boundary terms.

V. CONCLUSION

We have computed the total time derivative interactions in the third order action for the comoving curvature perturbation in general k-inflation. In previous calculations of the bispectrum these boundary terms have been ignored. In this note we have shown explicitly that a priori they are important and should not be neglected freely. These boundary interactions are necessary to erase the terms with second-order time derivatives on ζ in the action, generated by the integrations by parts. Total spatial derivative terms can be safely ignored because their contribution for the bispectrum is proportional to the sum of the three momentum vectors which has to be zero due to momentum conservation.

From all the boundary terms that appear after many integrations by parts in the action only one of them gives a non-zero bispectrum at leading order in the slow-variation expansion. We have shown that the bispectrum produced by this term is equal to the bispectrum produced in the usual field redefinition prescription, thus our results agree with previous results in the literature.

The main conclusion of this note is that in the calculation of the bispectrum in general k-inflation one can ignore all the boundary terms that appear when one simplifies the action but then one has to perform a field redefinition to eliminate terms in the action that are proportional to the first order equation of motion. On the other hand, one might choose to keep all the boundary terms and calculate the bispectrum using the usual method without the need to do the field redefinition. We have shown that in the end the bispectrum of the curvature perturbation is the same in both procedures.

An important lesson in computing the tree-level bispectrum is that we should basically use the reduced action written in terms of the physical variables that does not contain second order time derivatives in total. If second order time derivatives are contained, they must be eliminated by integration by parts. Only when the action takes the canonical form without second order time derivatives, one can use the expression for the third order action as it is in the path integral expression.

Note added: While we were writing up this work, the paper [67] appeared in the arXiv. They cite a paper in preparation [68] where the authors also argue that the inclusion of the boundary terms accounts for the terms introduced by the field redefinition.

Note added in version 2: Soon after this paper appeared on the arXiv, Ref. [68] appeared as [69]. Their section 3.1.2 contains similar, but independent, arguments on the role of the boundary terms. Some time after that, Ref. [70] appeared on-line and it also discusses the relation between field redefinitions, boundary terms and gauge transformations in the computation of the bispectrum.

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